

PREFACE I

Highlights of Gerry Ladas's biography

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Professor Gerry Ladas is a prolific writer and a highly cited mathematician. He has over 270 publications, including six research monographs, two undergraduate textbooks and has co-edited the proceedings of eight international conferences. He is also the major professor of 21 Ph.D. students.

Professor Ladas has co-founded with Professor Saber Elaydi, the *Journal of Difference Equations and Applications*, and they are the editors-in-chief of it. Professor Ladas is also an associate editor of 13 other mathematics journals.

Professor Ladas has an honorary doctorate degree from the University of Ioannina, Greece.

Professor Ladas has received both the excellence in teaching award and the excellence in research award from the University of Rhode Island. In the history of the University of Rhode Island, only two other professors have received both of these awards.

Professor Ladas has done seminal work in the oscillation theory of delay differential equations and difference equations. His monograph (with Györi) [7] is a highly cited monograph with 276 citations in MathSciNet.

Professor Ladas is an outstanding speaker and his talks at international meetings are always well organized and clear. His presentations on open problems and conjectures at the annual meetings of the ISDEA are among the highlights of the meetings.

Professor Gerry Ladas edits the section of the *Journal of Difference Equations and Applications* on open problems and conjectures. These open problems and conjectures have stimulated a substantial amount of interest, especially among young mathematicians and many research publications have been initiated from these open problems and conjectures.

Professor Gerry Ladas has made outstanding research contributions in both differential and difference equations and his work has been highly cited. Actually, in October 2007, he was selected to appear on ISIHighlyCited.com, because of his exceptional citation count in the field of Mathematics. His contributions to mathematics are evidenced by the high number of citations, his publications have received from fellow mathematicians. Less than one-half of 1% of all publishing authors meet the criteria for inclusion on ISIHighlyCited.com.

Professor Ladas has contributed a substantial amount of research work in the development of the basic theory of nonlinear difference equations. The following results are examples of his contributions:

THEOREM 1 (*EL-METWALLY ET AL.* [4,5]).

Let I be an interval of real numbers and let $F \in C(I^{k+1}, I)$. Assume that the following three conditions are satisfied:

- (1) *F is increasing in each of its arguments.*

- (2) $F(z_1, \dots, z_{k+1})$ is strictly increasing in each of the arguments $z_{i_1}, z_{i_2}, \dots, z_{i_l}$, where $1 \leq i_1 < i_2 < \dots < i_l \leq k+1$ and the arguments i_1, i_2, \dots, i_l are relatively prime.
- (3) Every point c in I is an equilibrium point of

$$x_{n+1} = F(x_n, x_{n-1}, \dots, x_{n-k}), \quad n = 0, 1, \dots \quad (1)$$

Then every solution of equation (1) has a finite limit.

THEOREM 2 (KOCIC AND LADAS [8]).

Assume that the following conditions hold:

- (i) $f \in C[(0, \infty) \times (0, \infty), (0, \infty)]$.
- (ii) $f(x, y)$ is decreasing in x and strictly decreasing in y .
- (iii) $xf(x, x)$ is strictly increasing in x .
- (iv) The equation

$$x_{n+1} = x_n f(x_n, x_{n-1}), \quad n = 0, 1, \dots \quad (2)$$

has a unique positive equilibrium \bar{x} .

Then \bar{x} is a global attractor of all positive solutions of equation (2).

THEOREM 3 (FRANKE ET AL. [6]). Assume that the following conditions hold:

- (i) $f \in C[[0, \infty) \times [0, \infty), (0, \infty)]$.
- (ii) $f(x, y)$ is decreasing in each argument.
- (iii) $xf(x, y)$ is increasing in x .
- (iv) $f(x, y) < f(y, x) \Leftrightarrow x > y$.
- (v) The equation

$$x_{n+1} = x_{n-1} f(x_{n-1}, x_n), \quad n = 0, 1, \dots$$

has a unique positive equilibrium \bar{x} .

Then \bar{x} is a global attractor of all positive solutions.

THEOREM 4 (KULENOVIC ET AL. [9], p. 201). Let $[a, b]$ be a closed and bounded interval of real numbers and let

$$F \in C([a, b]^{k+1}, [a, b]),$$

satisfy the following conditions:

- (1) The function $F(z_1, \dots, z_{k+1})$ is monotonic in each of its arguments.
- (2) For each $m, M \in [a, b]$ and for each $i \in \{1, \dots, k+1\}$, we define

$$M_i(m, M) = \begin{cases} M, & \text{if } F \text{ is increasing in } z_i, \\ m, & \text{if } F \text{ is decreasing in } z_i, \end{cases}$$

and

$$m_i(m, M) = M_i(M, m),$$

and we assume that if (m, M) is a solution of the system:

$$\left. \begin{aligned} M &= F(M_1(m, M), \dots, M_{k+1}(m, M)) \\ m &= F(m_1(m, M), \dots, m_{k+1}(m, M)) \end{aligned} \right\},$$

then $M = m$.

Then there exists exactly one equilibrium \bar{x} of equation (1) and every solution of equation (1) converges to \bar{x} .

THEOREM 5 (CAMOUZIS AND LADAS [3]). Assume that

$F \in C([0, \infty)^k, [0, \infty))$, $F(z_1, \dots, z_k)$ is monotonic in each of its arguments, and for each $m \in [0, \infty)$ and $M > m$, we assume that

$$F(M_1, \dots, M_k) \geq M \Rightarrow F(m_1, \dots, m_k) > m. \quad (3)$$

Then every solution of equation (1), which is bounded from above, converges to a finite limit.

THEOREM 6 (CAMOUZIS AND LADAS [3], p. 11 or REF. [2]).

Let I be a set of real numbers and let

$$F : I \times I \rightarrow I,$$

be a function $F(u, v)$, which decreases in u and increases in v . Then for every solution $\{x_n\}_{n=-1}^{\infty}$ of the equation

$$x_{n+1} = F(x_n, x_{n-1}), \quad n = 0, 1, \dots \quad (4)$$

the subsequences $\{x_{2n}\}_{n=0}^{\infty}$ and $\{x_{2n+1}\}_{n=-1}^{\infty}$ of even and odd terms are eventually monotonic.

THEOREM 7 (AMLEH ET AL. [1]).

Let I be a set of real numbers and let

$$F : I \times I \rightarrow I,$$

be a function $F(u, v)$ which increases in both variables. Then for every solution $\{x_n\}_{n=-1}^{\infty}$ of equation (4), the subsequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ of even and odd terms are eventually monotonic.

It is interesting to note that Theorems 6 and 7 can be extended to periodically forced difference equations as the following theorem states. See [2].

THEOREM 8 Let I be a set of real numbers and let

$$f_n : I \times I \rightarrow I$$

be an m -periodic sequence of functions $f_n(z_1, z_2)$, which are all increasing in z_2 and they are either all increasing or all decreasing in z_1 , throughout I . Then for every solution $\{x_n\}_{n=-1}^{\infty}$ of the difference equation

$$x_{n+1} = f_n(x_n, x_{n-1}), \quad n = 0, 1, \dots$$

The $2m$ -sequences $\{x_{2mn+t}\}_{t=0}^{2m-1}$ are eventually monotonic when m is odd, and when m is even, the m -sequences $\{x_{mn+t}\}_{t=0}^{m-1}$ are eventually monotonic.

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